Identifying the quark content of the isoscalar scalar mesons $f_0(980)$, $f_0(1370)$, and $f_0(1500)$ from weak and electromagnetic processes

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Abstract

The assignments of the isoscalar scalar mesons $f_0(980)$, $f_0(1370)$, and $f_0(1500)$ in terms of their $\bar{q}q$ substructure is still a matter of heated dispute. Here we employ the weak and electromagnetic decays $D_s^+ \to f_0\pi^+$ and $f_0 \to \gamma\gamma$, respectively, to identify the $f_0(980)$ and $f_0(1500)$ as mostly $\bar{s}s$, and the $f_0(1370)$ as dominantly $\bar{n}n$, in agreement with previous work. The two-photon decays can be satisfactorily described with quark as well as with meson loops, though the latter ones provide a less model-dependent and more quantitative description.

1 Introduction

A proper classification of the scalar mesons is still being clouded by two major problems, which mutually hamper the resolution of either. The first difficulty is the apparent excess of experimentally confirmed scalar resonances with respect to the number of theoretically expected $\bar{q}q$ states. The second problem is to unambiguously identify the $\bar{q}q$ configuration of the isoscalar

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scalar mesons, i.e., the $f_0(400\text{–}1200)$ (or σ), $f_0(980)$, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$. In previous work, we have especially addressed the former issue, showing that the light (below 1 GeV) scalars can be described as a complete nonet of $\bar{q}q$ states, resulting from either the dynamical breaking of chiral symmetry [1], or the coupling of bare P-wave $\bar{q}q$ systems to the meson-meson continuum in a unitarized approach [2, 3]. We believe that these two mechanisms are intimately related to one another, though in a not yet completely understood fashion. In any case, in both pictures the scalar mesons between 1.3 and 1.5 GeV form another nonet, and so forth. So we conclude there is no excess of observed states, thus dispensing with the introduction of new degrees of freedom like multiquark states, glueballs, $\bar{K}K$ molecules, and so on.

Here, we want to focus on the second issue, namely the identification of the isoscalars, especially the vehemently disputed $f_0(980)$, $f_0(1370)$, and $f_0(1500)$, in an as model-independent way as one may achieve. In Refs. [4, 5] we have already presented qualitative arguments from observed hadronic decays that favor, in our view, a mainly $\bar{s}s$ configuration for the $f_0(980)$ and $f_0(1500)$, and a dominantly nonstrange $\bar{q}q$ content for the $f_0(1370)$. Furthermore, we are engaged in substantiating these arguments by analysing also the four-pion decays of these scalars via intermediate $\rho\rho$ and $\sigma\sigma$ two-resonance states, in a similar way as done for the $\omega \to \rho\pi \to \pi\pi\pi$ cascade process in Ref. [6]. In the present work, we shall employ the weak and electromagnetic decays (as opposed to the more complicated strong-interaction dynamics) $D_s^+ \to f_0\pi^+$ and $f_0 \to \gamma\gamma$, respectively, which will give quantitative support for our $\bar{q}q$ assignments. These processes will be analysed in a simple $\bar{q}q$ picture for the corresponding f_0 resonances, with a minimum of model-dependent input.

This paper is organized as follows. In Section 2 we compute the weak decays $D_s^+ \to \pi^+ f_0(980)$, $\pi^+ f_0(1500)$, $\pi^+ f_0(1710)$ using W^+ emission. In Section 3 we calculate the $f_0(980)$, $f_0(1370) \to 2\,\gamma$ electromagnetic decays, employing quark as well as meson loops. Conclusions are drawn in Section 4.

2 Weak decays $D_s^+ \to \pi^+ f_0$

First we compute the parity-conserving weak decays $D_s^+ \to \pi^+ f_0(980)$ and $\pi^+ f_0(1500)$, supposing for the moment that both of these final-state scalar mesons are purely $\bar{s}s$. Given the Fermi Hamiltonian density $H_W = \frac{G_F}{2\sqrt{2}}\,(J\,J^+ + J^+ J)$ with [9] $G_F = 1.16639(1)\cdot 10^{-5}$ GeV^{-2} and $F_\pi = f_{\pi^+}/\sqrt{2} \simeq (92.42\pm0.27)$ MeV, the magnitudes of the corresponding weak decay amplitudes of W^+ emission are [7] (also see Ref. [8])

$$|M(D_s^+ \to \pi^+ f_0(980))| \;=\; \frac{G_F\,|V_{ud}|\,|V_{cs}|}{2}\,F_\pi\,(m^2_{D_s^+} - m^2_{f_0(980)})$$

$$= (159 \pm 24) \cdot 10^{-8}\ \mathrm{GeV}, \tag{1}$$

$$|M(D_s^+ \to \pi^+ f_0(1500))| = \frac{G_F\,|V_{ud}|\,|V_{cs}|}{2}\, F_\pi\, (m^2_{D_s^+} - m^2_{f_0(1500)})$$

$$= (89 \pm 13) \cdot 10^{-8}\ \mathrm{GeV}, \tag{2}$$

being both close to the data [9] $(178{\pm}40){\cdot}10^{-8}$ GeV and $(96{\pm}28){\cdot}10^{-8}$ GeV, respectively. The latter amplitudes are extracted from the observed decay rates Γ according to $|M| = m_{D_s^+}\sqrt{8\pi\,\Gamma/q_{cm}}$. The agreement of Eqs. (1) and (2) with the data has already been noted in Refs. [10] and [5], respectively.

Another way to study Eqs. (1) and (2) above is to take the ratio

$$\left|\frac{M(D_s^+ \to \pi^+ f_0(980))}{M(D_s^+ \to \pi^+ f_0(1500))}\right|_{|f_0>=|\bar{s}s>} = \frac{m^2_{D_s^+} - m^2_{f_0(980)}}{m^2_{D_s^+} - m^2_{f_0(1500)}} = 1.79 \pm 0.04, \tag{3}$$

(using $m_{f_0(980)} = (980 \pm 10)$ MeV, $m_{f_0(1500)} = (1500 \pm 10)$ MeV and $m_{D_s^+} = (1968.6 \pm 0.6)$ MeV), which is independent of the weak scale G_F, the CKM parameters $|V_{ud}|$, $|V_{cs}|$, and the pion decay constant F_π. As such, Eq. (3) is the kinematic (model-independent) infinite-momentum-frame (IMF) (see e.g. Ref. [11]) version. The data [9] depend on the branching ratio and center-of-mass (CM) momenta as

$$\left|\frac{M(D_s^+ \to \pi^+ f_0(980))}{M(D_s^+ \to \pi^+ f_0(1500))}\right|_{\mathrm{PDG}} = \sqrt{\frac{\Gamma(D_s^+ \to \pi^+ f_0(980))\ \ q_{cm}(D_s^+ \to \pi^+ f_0(1500))}{\Gamma(D_s^+ \to \pi^+ f_0(1500))\ \ q_{cm}(D_s^+ \to \pi^+ f_0(980))}} = 1.86{\pm}0.68\ , \tag{4}$$

showing again a very good agreement. Here, we have used the measured branching ratios [9] $\Gamma(D_s^+ \to \pi^+ f_0(980))/\Gamma(D_s^+) = (1.8 \pm 0.8)\%$ and $\Gamma(D_s^+ \to \pi^+ f_0(1500))/\Gamma(D_s^+) = (0.28 \pm 0.16)\%$, and the corresponding extracted CM momenta $q_{cm}(D_s^+ \to \pi^+ f_0(980)) = (732.1 \pm 5.1)$ MeV/c and $q_{cm}(D_s^+ \to \pi^+ f_0(1500)) = (393.8 \pm 8.1)$ MeV/c. The large error $\pm\,0.68$ in Eq. (4) stems from the uncertainties in the measured branching ratios, rather than from the quite accurately known CM momenta. These uncertainties leave quite some room to allow for significant $\bar{n}n$ admixtures in the $f_0(980)$ as well as the $f_0(1500)$, without calling into question their $\bar{s}s$ dominance. On the other hand, from the failure to observe the decay $D_s^+ \to \pi^+ f_0(1370)$ [9] (see however Ref. [12]) it seems safe to conclude that the $f_0(1370)$ does not have a large $\bar{s}s$ component.

To conclude the weak processes, let us look at the situation for the $f_0(1710)$. Although the weak decay $D_s^+ \to \pi^+ f_0(1710)$ has been observed, the quoted rate $(1.5 \pm 1.9) \times 10^{-3}$ [9], corresponding to an amplitude of $(97 \pm 123) \cdot 10^{-8}$ GeV, only accounts for K^+K^- decays of this resonance. The theoretical W^+-emission amplitude has a magnitude of $52 \cdot 10^{-8}$ GeV. In view of the huge experimental error, no definite conclusions on the $\bar{q}q$ (or any other) substructure of the $f_0(1710)$ are possible for the time being. Nevertheless, the sheer observation of the weak decay process seems to preclude a dominantly $\bar{n}n$ configuration. Indeed, the Meson Particle Listings conclude that the $f_0(1710)$ *"is consistent with a large $\bar{s}s$ component"* (Ref. [9], page 470).

3 Electromagnetic scalar decays $S \to 2\,\gamma$

3.1 The decay $f_0(980) \to 2\,\gamma$

The PDG tables [9] now reports the scalar $f_0(980) \to 2\,\gamma$ decay rate as (0.39 ± 0.12) keV. Given the scalar amplitude structure [13, 14, 15, 16] $M\,\varepsilon_\mu(k')\,\varepsilon_\nu(k)\,(g^{\mu\nu}k'\cdot k - k'^\mu k^\nu)$, the $S \to 2\,\gamma$ decay rate is

$$\Gamma(f_0 \to 2\,\gamma) = \frac{m_{f^0}^3\,|M|^2}{64\,\pi}\,, \quad \text{or} \quad |M(f_0(980) \to 2\,\gamma)| = (0.91 \pm 0.14) \cdot 10^{-2}\ \text{GeV}^{-1} \tag{5}$$

If the $f_0(980)$ were $\bar{n}n$, the isoscalar u,d quark-loop analogue of the isovector $\pi^0 \to 2\,\gamma$ amplitude, given by [14] $\sqrt{2}\,\alpha\,N_c\,\mathrm{Tr}\,[Q^2\,Q_{\bar{n}n}]\,/\,(\pi\,F_\pi) = 5\,\alpha\,N_c\,/\,(9\,\pi\,F_\pi) \simeq 0.042\ \text{GeV}^{-1}$ with $N_c = 3$, would generate an $f_0(980) \to 2\,\gamma$ decay rate a factor of <u>21 times too large</u> [1]. If, instead, the $f_0(980)$ is a pure $\bar{s}s$ state, the $f_0 \to 2\,\gamma$ amplitude magnitude becomes [14] $\alpha\,N_c\,g_{f_0\,SS}\,/(9\,\pi\,m_s) \simeq 0.81 \cdot 10^{-2}$ GeV^{-1}, using $g_{f_0\,SS} = \sqrt{2}\ 2\pi/\sqrt{3}$ and constituent strange quark mass [17, 1] $m_s = 490$ MeV $\simeq 1.44\,\hat{m}$ (from Ref. [17], $F_K/F_\pi = (\hat{m}+m_s)/(2\,\hat{m}) \simeq 1.22$) with the constituent nonstrange mass $\hat{m} \simeq 340$ MeV. This value lies reasonably close the observed amplitude in Eq. (5).[2] However, at this point we should note that the quark-loop result for the two-photon decay rate is very sensitive to a possible $\bar{n}n$ admixture in the $f_0(980)$, due to an enhancement factor of 25 of the $\bar{n}n$ component with respect to the $\bar{s}s$ component. This factor comes from the electric charge of the quarks, yielding $((\frac{2}{3})^2 + (\frac{1}{3})^2)^2$ for the nonstrange isoscalar $\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$, and $(\frac{1}{3})^4$ for the strange isoscalar.

Therefore, rather than involving the model-dependent quark coupling and constituent quark masses as above, we instead consider a combination of the decay chains $f_0 \to K^+K^- \to 2\,\gamma$ and $f_0 \to \pi^+\pi^- \to 2\,\gamma$ [13, 14, 15, 16]. According to Refs. [13, 16], the kaon loop is suppressed by 10% due to a, so far experimentally unconfirmed, scalar $\kappa(900)$. (However, very recent results from

[1]We introduced the SU(3) charge matrix $Q = T_3 + Y/2 = \mathrm{Diag}\,[2/3, -1/3, -1/3] = (\lambda_3 + \lambda_8/\sqrt{3}\,)/2$ and the $\bar{n}n = (\bar{u}u + \bar{d}d)/\sqrt{2}$ analogue $Q_{\bar{n}n}$ =Diag $[1/\sqrt{2}, 1/\sqrt{2}, 0] = (\lambda_0 + \lambda_8/\sqrt{2}\,)/\sqrt{3}$.

[2]Without changes, we could of course also use the identity $\sqrt{2}\,\alpha\,N_c\,\mathrm{Tr}\,[Q^2\,Q_{\bar{s}s}]\,/\,(\pi\,F_{\bar{s}s}) = \sqrt{2}\,\alpha\,N_c\,/\,(9\,\pi\,F_{\bar{s}s}) \simeq 0.81 \cdot 10^{-2}$ GeV^{-1}, with $F_{\bar{s}s} = \sqrt{3}\,m_s/(2\,\pi) = 135.1$ MeV $\simeq 1.2\,F_K \simeq 2F_K - F_\pi \simeq \sqrt{2}\,F_\pi$ and $Q_{\bar{s}s}$ =Diag $[0, 0, 1] = (\lambda_0/\sqrt{2} - \lambda_8\,)/\sqrt{3}$. The use of [9] $F_K = f_{K^+}/\sqrt{2} = (113.00 \pm 1.04)$ MeV instead of $F_{\bar{s}s}$ would bring us even closer to the data, as $\sqrt{2}\,\alpha\,N_c\,/\,(9\,\pi\,F_K) \simeq 0.972 \cdot 10^{-2}$ GeV^{-1}.

the E791 collaboration present preliminary evidence for a light κ (see e-print in Ref. [12]), which would confirm our prediction [1, 2] of such a state.) In order to proceed, we have to remind the reader to the standard mixing scheme between the "physical" states ($|\sigma(600)>$ and $|f_0(980)>$), and the nonstrange and strange basis states $|\bar{n}n>$ and $|\bar{s}s>$, i.e.,

$$\begin{aligned}|\sigma(600)> &= \cos\phi_s\,|\bar{n}n> -\, \sin\phi_s\,|\bar{s}s>\,, \\ |f_0(980)> &= \sin\phi_s\,|\bar{n}n> +\, \cos\phi_s\,|\bar{s}s>\ . \end{aligned} \tag{6}$$

With quadratic mass mixing, one can define for the states $|\bar{n}n>$ and $|\bar{s}s>$ the nonstrange and strange mass parameters $m_{\bar{n}n}$ and $m_{\bar{s}s}$ by [14] as

$$\begin{aligned} m^2_{\bar{n}n} &= \cos^2\phi_s\, m^2_\sigma \,+\, \sin^2\phi_s\, m^2_{f_0} \;=\; \Big((646 \pm 10)\ \mathrm{MeV}\Big)^2 , \\ m^2_{\bar{s}s} &= \sin^2\phi_s\, m^2_\sigma \,+\, \cos^2\phi_s\, m^2_{f_0} \;=\; \Big((950 \pm 11)\ \mathrm{MeV}\Big)^2 . \end{aligned} \tag{7}$$

Throughout this paper we choose a mixing angle of [3] $\phi_s \simeq 18^\circ \pm 2^\circ$ [1, 18, 17, 14] or $\phi_s \simeq -(18^\circ \pm 2^\circ)$ [16], and assume the scalar-meson masses to be $m_{f_0(980)} = (980 \pm 10)$ MeV [9] and $m_{\sigma(600)} = 600$ MeV. Since the interaction Lagrangians between the f_0 and the pseudoscalars $\pi^\pm$ and $K^\pm$ are proportional to f_0, the Lagrangians can, within the same mixing scheme, be simultaneously reexpressed in terms of nonstrange and strange fields, i.e.,

$$\begin{aligned} &\mathcal{L}\,(f_0\pi\pi) + \mathcal{L}\,(f_0KK) = \\ &\quad = \ \sin\phi_s\,\Big(\mathcal{L}\,(\bar{n}n\,\pi\pi) + \mathcal{L}\,(\bar{n}n\,KK)\Big) \ + \ \cos\phi_s\,\Big(\mathcal{L}\,(\bar{s}s\,\pi\pi) + \mathcal{L}\,(\bar{s}s\,KK)\Big) \end{aligned} \tag{8}$$

Within the usual nonet, that is, the U(3) picture, the scalar (S) and pseudoscalar (P) fields are proportional to linear combinations of the Gell-Mann matrices $\lambda_0, \lambda_1, \ldots, \lambda_8$ (λ_0 denotes here $\sqrt{2/3}\ 1_3$ with 1_3 being the 3-dimensional unit matrix), denoted by Q_S and Q_P, respectively. From the quark content of the corresponding mesonic systems, it is easy to derive

$$\begin{aligned} \bar{n}\,n = \frac{1}{\sqrt{2}}\,(\bar{u}u + \bar{d}d) \ &\Rightarrow\ Q_{\bar{n}\,n} = \frac{1}{\sqrt{3}}\left(\lambda_0 + \frac{1}{\sqrt{2}}\,\lambda_8\right)\,, \\ \bar{s}\,s \ &\Rightarrow\ Q_{\bar{s}\,s} = \frac{1}{\sqrt{3}}\left(\frac{1}{\sqrt{2}}\,\lambda_0 - \lambda_8\right)\,, \\ \\ \pi^+ = \bar{d}u\ ,\ \pi^- = \bar{u}d \ &\Rightarrow\ Q_{\pi^\pm} = \frac{1}{2}\,(\lambda_1 \pm i\,\lambda_2)\ , \\ K^+ = \bar{s}u\ ,\ K^- = \bar{u}s \ &\Rightarrow\ Q_{K^\pm} = \frac{1}{2}\,(\lambda_4 \pm i\,\lambda_5)\ . \end{aligned} \tag{9}$$

[3] The sign of the mixing angle, which cannot be identified from a quadratic mass mixing scheme, has still to be determined from theoretical consistency arguments, as it has a strong influence on the interference terms in the present work.

In the linear σ model (LSM), the interaction Lagrangian $\mathcal{L}(S\,P_1\,P_2)$ is proportional to the flavor trace Tr $(Q_S\,\{Q_{P_1}, Q_{P_2}\})$, and so are the corresponding coupling constants. It should be mentioned that the charge of a mesonic system ϕ is determined by Tr $(Q\,[Q_\phi, Q_\phi^T])$. Thus, we derive for the relevant channels under consideration, i.e., $\bar{n}n \to \pi\pi$, $\bar{n}n \to KK$, $\bar{s}s \to \pi\pi$, and $\bar{s}s \to KK$:

$$
\begin{aligned}
d_{\bar{n}n\,\pi^+\pi^-} &= \frac{1}{\sqrt{2}}\,\mathrm{Tr}\,(Q_{\bar{n}n}\,\{Q_{\pi^+}, Q_{\pi^-}\}) &= 1\,, \\
d_{\bar{n}n\,K^+K^-} &= \frac{1}{\sqrt{2}}\,\mathrm{Tr}\,(Q_{\bar{n}n}\,\{Q_{K^+}, Q_{K^-}\}) &= \frac{1}{2}\,, \\
d_{\bar{s}s\,\pi^+\pi^-} &= \frac{1}{\sqrt{2}}\,\mathrm{Tr}\,(Q_{\bar{s}s}\,\{Q_{\pi^+}, Q_{\pi^-}\}) &= 0\,, \\
d_{\bar{s}s\,K^+K^-} &= \frac{1}{\sqrt{2}}\,\mathrm{Tr}\,(Q_{\bar{s}s}\,\{Q_{K^+}, Q_{K^-}\}) &= \frac{1}{\sqrt{2}}\,.
\end{aligned}
\tag{10}
$$

The corresponding equivalent symmetric structure constants $d_{\bar{n}n\,33}$, $d_{\bar{n}n\,K^0K^0}$, $d_{\bar{s}s\,33}$, $d_{\bar{s}s\,K^0K^0}$, with $d_{abc} = \mathrm{Tr}(\lambda_a\{\lambda_b, \lambda_c\})/4$, for two neutral pseudoscalars in the final state have already been derived in Ref. [17]. In accordance with the σ-model results, we determine the corresponding SU(3) couplings for $\phi_s \simeq +(18^\circ \pm 2^\circ)$ and $\phi_s \simeq -(18^\circ \pm 2^\circ)$ as

$$
\begin{aligned}
g'_{\bar{n}n\,\pi\pi} &= d_{\bar{n}n\,\pi^+\pi^-}\,\frac{m^2_{\bar{n}n} - m^2_{\pi^\pm}}{2\,F_\pi} = \frac{\cos^2\phi_s\,m^2_\sigma\,+\,\sin^2\phi_s\,m^2_{f_0} - m^2_{\pi^\pm}}{2\,F_\pi} \\
&= (2.152 \pm 0.068)\ \mathrm{GeV}\,, \\
g'_{\bar{n}n\,KK} &= d_{\bar{n}n\,K^+K^-}\,\frac{m^2_{\bar{n}n} - m^2_{K^\pm}}{F_K} = \frac{\cos^2\phi_s\,m^2_\sigma\,+\,\sin^2\phi_s\,m^2_{f_0} - m^2_{K^\pm}}{2\,F_K} \\
&= (0.768 \pm 0.056)\ \mathrm{GeV}\,, \\
g'_{\bar{s}s\,\pi\pi} &= d_{\bar{s}s\,\pi^+\pi^-}\,\frac{m^2_{\bar{s}s} - m^2_{\pi^\pm}}{2\,F_\pi} = 0\,, \\
g'_{\bar{s}s\,KK} &= d_{\bar{s}s\,K^+K^-}\,\frac{m^2_{\bar{s}s} - m^2_{K^\pm}}{F_K} = \frac{\sin^2\phi_s\,m^2_\sigma\,+\,\cos^2\phi_s\,m^2_{f_0} - m^2_{K^\pm}}{\sqrt{2}\,F_K} \\
&= (4.126 \pm 0.141)\ \mathrm{GeV}\,,
\end{aligned}
\tag{11}
$$

yielding for $\phi_s \simeq +(18^\circ \pm 2^\circ)$

$$
\begin{aligned}
(\sin\phi_s\ \ g'_{\bar{n}n\,\pi\pi}\ +\ \cos\phi_s\ g'_{\bar{s}s\,\pi\pi}) &= (0.665 \pm 0.093)\ \mathrm{GeV}\,, \\
(\sin\phi_s\ g'_{\bar{n}n\,KK}\ + \cos\phi_s\ g'_{\bar{s}s\,KK}) &= (4.162 \pm 0.138)\ \mathrm{GeV}\,,
\end{aligned}
\tag{12}
$$

and for $\phi_s \simeq -(18^\circ \pm 2^\circ)$

$$
\begin{aligned}
(\sin\,\phi_s\;\; g'_{\bar n n\,\pi\pi}\;\; +\;\cos\,\phi_s\; g'_{\bar s s\,\pi\pi}) \;\; &= \;\; (-\,0.665 \pm 0.093)\ \mathrm{GeV}\,, \\
(\sin\,\phi_s\; g'_{\bar n n\,KK}\; +\cos\,\phi_s\; g'_{\bar s s\,KK}) \;\; &= \;\; (3.687 \pm 0.194)\ \mathrm{GeV}\,.
\end{aligned} \qquad (13)
$$

In order to compute these numbers, we used $F_\pi \simeq (92.42 \pm 0.27)$ MeV, $F_K \simeq (113.00 \pm 1.04)$ MeV, i.e. $F_K/F_\pi \simeq 1.22$. Putting all this together, we obtain for the pion- and kaon-loop amplitudes [13]

$$
\begin{aligned}
M_{\pi\text{-loop}} &= \frac{2\,\alpha\,(\sin\,\phi_s\; g'_{\bar n n\,\pi\pi}\; +\cos\,\phi_s\; g'_{\bar s s\,\pi\pi})}{\pi\, m^2_{f_0}} \left[-\frac{1}{2} + \xi_\pi\, I(\xi_\pi)\right] \\
&= \;\; (-0.177 \pm 0.025 + i\,(+\,0.079 \pm 0.012)) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for} \;\; \phi_s \simeq +\,(18^\circ \pm 2^\circ) \\
&= \;\; (+0.177 \pm 0.025 + i\,(-\,0.079 \pm 0.012)) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for} \;\; \phi_s \simeq -\,(18^\circ \pm 2^\circ)\,,
\end{aligned}
$$

$$
\begin{aligned}
M_{\text{K-loop}} &= \frac{2\,\alpha\,(\sin\,\phi_s\; g'_{\bar n n\,KK}\; +\cos\,\phi_s\; g'_{\bar s s\,KK})}{\pi\, m^2_{f_0}} \left[-\frac{1}{2} + \xi_K\, I(\xi_K)\right] \\
&= \;\; (1.138 \pm 0.254) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for} \;\; \phi_s \simeq +\,(18^\circ \pm 2^\circ) \\
&= \;\; (1.008 \pm 0.229) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for} \;\; \phi_s \simeq -\,(18^\circ \pm 2^\circ)\,,
\end{aligned}
$$

$$
\begin{aligned}
& M_{\pi\text{-loop}} + M_{\text{K-loop}} = \\
&= \;\; (0.960 \pm 0.255 + i\,(+\,0.079 \pm 0.012)) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for} \;\; \phi_s \simeq +\,(18^\circ \pm 2^\circ) \\
&= \;\; (1.185 \pm 0.230 + i\,(-\,0.079 \pm 0.012)) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for} \;\; \phi_s \simeq -\,(18^\circ \pm 2^\circ)\,,
\end{aligned}
$$

$$
\begin{aligned}
& |M_{\pi\text{-loop}} + M_{\text{K-loop}}| = \\
&= \;\; (0.964 \pm 0.255) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for} \;\; \phi_s \simeq +\,(18^\circ \pm 2^\circ) \\
&= \;\; (1.188 \pm 0.230) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for} \;\; \phi_s \simeq -\,(18^\circ \pm 2^\circ)\,.
\end{aligned} \qquad (14)
$$

As $\xi_\pi = m^2_{\pi^+}/m^2_{f_0(980)} = 0.02028 \pm 0.00042 < 1/4$, the value of the pion-loop integral is obtained from (see also p. 230, 422 in Ref. [19])

$$
\begin{aligned}
I(\xi_\pi) \;\; &= \;\; \int_0^1 dy \int_0^1 dx\; \frac{y}{\xi_\pi - xy\,(1-y)} \;\; = \;\; 2\left[\frac{\pi}{2} + i\,\ln\left(\sqrt{\frac{1}{4\,\xi_\pi}} + \sqrt{\frac{1}{4\,\xi_\pi} - 1}\right)\right]^2 \\
&= \;\; \frac{\pi^2}{2} - 2\,\ln^2\left[\sqrt{\frac{1}{4\,\xi_\pi}} + \sqrt{\frac{1}{4\,\xi_\pi} - 1}\,\right] + 2\,\pi\, i\,\ln\left[\sqrt{\frac{1}{4\,\xi_\pi}} + \sqrt{\frac{1}{4\,\xi_\pi} - 1}\,\right] \\
&= \;\; -\,2.500 \pm 0.083 + i\,(12.114 \pm 0.067)\,,
\end{aligned}
$$

while, as $\xi_K = m_{K^+}^2/m_{f_0(980)}^2 = 0.2538 \pm 0.0052 > 1/4$, the kaon loop follows from

$$I(\xi_K) \;=\; \int_0^1 dy \int_0^1 dx\; \frac{y}{\xi_K - xy\,(1-y)} \;=\; 2\left[\arcsin\sqrt{\frac{1}{4\,\xi_K}}\;\right]^2 = 4.197 \pm 0.482\,, \qquad (15)$$

yielding, respectively,

$$\begin{aligned} -\frac{1}{2} + \xi_\pi\, I(\xi_\pi) &= -\,0.5507 \pm 0.0020 + i\,(0.2457 \pm 0.0037)\,, \\ -\frac{1}{2} + \xi_K\, I(\xi_K) &= 0.5651 \pm 0.1242\,. \end{aligned} \qquad (16)$$

Reducing the kaon-loop amplitude in Eq. (14) by 10 % (owing to the scalar $\kappa(900)$ loop), but leaving the value of its error unaltered, predicts $(0.85 \pm 0.26)\cdot 10^{-2}$ GeV^{-1} ($\phi_s \simeq +(18^\circ \pm 2^\circ)$) or $(1.09 \pm 0.23)\cdot 10^{-2}$ GeV^{-1} ($\phi_s \simeq -(18^\circ \pm 2^\circ)$) for the modulus of the $f_0(980) \to 2\,\gamma$ amplitude, reasonably near the data [9] in Eq. (5). Therefore, whether we employ quark loops or instead π and K loops as in Eq. (14), it is clear that the $f_0(980) \to 2\,\gamma$ amplitude can only be understood, if the $f_0(980)$ is mostly $\bar{s}s$.[4] This is the same conclusion as obtained, more easily, from the weak decay $D_s^+ \to \pi^+ f_0(980)$ in Eq. (1).

3.2 The decay $f_0(1370) \to 2\,\gamma$

Now we study the process $f_0(1370) \to 2\,\gamma$, using the same techniques as above. The observed decay rate [9] is about (4.6 ± 2.8) keV, with amplitude given by (using $m_{f_0(1370)} = (1370 \pm 170)$ MeV)

$$\Gamma(f_0 \to 2\,\gamma) = \frac{m_{f^0}^3\,|M|^2}{64\,\pi}\,, \quad \text{or} \quad |M(f_0(1370) \to 2\,\gamma)| = (1.90 \pm 0.68)\cdot 10^{-2}\ \text{GeV}^{-1}\,. \qquad (17)$$

In order to apply again a meson-loop approach, we develop once more a meson-mixing scheme, namely

$$\begin{aligned} |f_0(1370)> &= \cos\phi_s'\;|\bar{n}n> \;-\; \sin\phi_s'\;|\bar{s}s> \\ |f_0(1500)> &= \sin\phi_s'\;|\bar{n}n> \;+\; \cos\phi_s'\;|\bar{s}s>\;. \end{aligned} \qquad (18)$$

Again we define, using quadratic mass mixing with respect to the states $|\bar{n}n>$ and $|\bar{s}s>$, the nonstrange and strange mass parameters $m'_{\bar{n}n}$ and $m'_{\bar{s}s}$ by

$$\begin{aligned} m'^2_{\bar{n}n} &= \cos^2\phi_s'\; m^2_{f_0(1370)} \;+\; \sin^2\phi_s'\; m^2_{f_0(1500)}\,, \\ m'^2_{\bar{s}s} &= \sin^2\phi_s'\; m^2_{f_0(1370)} \;+\; \cos^2\phi_s'\; m^2_{f_0(1500)}\,. \end{aligned} \qquad (19)$$

[4] Surely, the error bars of the presented analysis rely strongly on the assumption that we choose a sharp σ-meson mass $m_\sigma = 600$ MeV, without any uncertainty.

Consequently, we use the couplings

$$
\begin{aligned}
g'_{\bar{n}n\,\pi\pi} &= d_{\bar{n}n\,\pi^+\pi^-}\ \frac{m'^2_{\bar{n}n} - m^2_{\pi^\pm}}{2\,F_\pi} = \frac{\cos^2\phi_s'\ m^2_{f_0(1370)}\ +\ \sin^2\phi_s'\ m^2_{f_0(1500)} - m^2_{\pi^\pm}}{2\,F_\pi} \\
&= (10.05 \pm 2.53)\ \text{GeV} \quad \text{for}\ \ \phi_s' = 0^\circ\,, \\
&= (10.24 \pm 2.29)\ \text{GeV} \quad \text{for}\ \ \phi_s' \simeq \pm\,(18^\circ \pm 2^\circ)\,, \\
g'_{\bar{n}n\,KK} &= d_{\bar{n}n\,K^+K^-}\ \frac{m'^2_{\bar{n}n} - m^2_{K^\pm}}{F_K} = \frac{\cos^2\phi_s'\ m^2_{f_0(1370)}\ +\ \sin^2\phi_s'\ m^2_{f_0(1500)} - m^2_{K^\pm}}{2\,F_K} \\
&= (7.23 \pm 2.07)\ \text{GeV} \quad \text{for}\ \ \phi_s' = 0^\circ\,, \\
&= (7.38 \pm 1.87)\ \text{GeV} \qquad \text{for}\ \ \phi_s' \simeq \pm\,(18^\circ \pm 2^\circ)\,, \\
g'_{\bar{s}s\,\pi\pi} &= d_{\bar{s}s\,\pi^+\pi^-}\ \frac{m'^2_{\bar{s}s} - m^2_{\pi^\pm}}{2\,F_\pi}\ =\ 0 \\
g'_{\bar{s}s\,KK} &= d_{\bar{s}s\,K^+K^-}\ \frac{m'^2_{\bar{s}s} - m^2_{K^\pm}}{F_K} = \frac{\sin^2\phi_s'\ m^2_{f_0(1370)}\ +\ \cos^2\phi_s'\ m^2_{f_0(1500)} - m^2_{K^\pm}}{\sqrt{2}\ F_K} \\
&= (12.56 \pm 0.23)\ \text{GeV} \quad \text{for}\ \ \phi_s' = 0^\circ\,, \\
&= (12.33 \pm 0.35)\ \text{GeV} \quad \text{for}\ \ \phi_s' \simeq \pm\,(18^\circ \pm 2^\circ)\,,
\end{aligned} \tag{20}
$$

yielding, respectively,

$$
\begin{aligned}
(\cos\phi_s'\ g'_{\bar{n}n\,\pi\pi}\ - \sin\phi_s'\ g'_{\bar{s}s\,\pi\pi}) &= (10.05 \pm 2.53)\ \text{GeV} \quad \text{for}\ \ \phi_s' = 0^\circ\,, \\
&= (9.74 \pm 2.17)\ \text{GeV} \quad \text{for}\ \ \phi_s' \simeq +\,(18^\circ \pm 2^\circ)\,, \\
&= (9.74 \pm 2.17)\ \text{GeV} \quad \text{for}\ \ \phi_s' \simeq -\,(18^\circ \pm 2^\circ)\,, \\
(\cos\phi_s'\ g'_{\bar{n}n\,KK}\ - \sin\phi_s'\ g'_{\bar{s}s\,KK}) &= (7.23 \pm 2.07)\ \text{GeV} \quad \text{for}\ \ \phi_s' = 0^\circ\,, \\
&= (3.21 \pm 1.75)\ \text{GeV} \quad \text{for}\ \ \phi_s' \simeq +\,(18^\circ \pm 2^\circ)\,, \\
&= (10.83 \pm 1.90)\ \text{GeV} \quad \text{for}\ \ \phi_s' \simeq -\,(18^\circ \pm 2^\circ)\,,
\end{aligned} \tag{21}
$$

to determine the pion- and kaon-loop amplitudes

$$
\begin{aligned}
M_{\pi\text{-loop}} &= \frac{2\,\alpha\,(\cos\phi_s'\ g'_{\bar{n}n\,\pi\pi}\ - \sin\phi_s'\ g'_{\bar{s}s\,\pi\pi})}{\pi\,m^2_{f_0(1370)}}\left[-\frac{1}{2} + \xi_\pi\,I(\xi_\pi)\right]\,, \\
M_{\text{K-loop}} &= \frac{2\,\alpha\,(\cos\phi_s'\ g'_{\bar{n}n\,KK}\ - \sin\phi_s'\ g'_{\bar{s}s\,KK})}{\pi\,m^2_{f_0(1370)}}\left[-\frac{1}{2} + \xi_K\,I(\xi_K)\right]\,.
\end{aligned} \tag{22}
$$

Using $\xi_\pi = m^2_{\pi^+}/m^2_{f_0(1370)} = 0.0104 \pm 0.0026 < 1/4$ and $\xi_K = m^2_{K^+}/m^2_{f_0(1370)} = 0.1299 \pm 0.0323 < 1/4$, we obtain [13] (see also p. 230, 422 in Ref. [19])

$$\begin{aligned}
I(\xi_\pi) &= \frac{\pi^2}{2} - 2\,\ln^2\left[\sqrt{\frac{1}{4\,\xi_\pi}} + \sqrt{\frac{1}{4\,\xi_\pi} - 1}\,\right] + 2\,\pi\,i\,\ln\left[\sqrt{\frac{1}{4\,\xi_\pi}} + \sqrt{\frac{1}{4\,\xi_\pi} - 1}\,\right] \\
&= -5.40 \pm 1.16 + i\,(14.28 \pm 0.80)\,, \\
I(\xi_K) &= \frac{\pi^2}{2} - 2\,\ln^2\left[\sqrt{\frac{1}{4\,\xi_K}} + \sqrt{\frac{1}{4\,\xi_K} - 1}\,\right] + 2\,\pi\,i\,\ln\left[\sqrt{\frac{1}{4\,\xi_K}} + \sqrt{\frac{1}{4\,\xi_K} - 1}\,\right] \\
&= 3.48 \pm 0.62 + i\,(5.37 \pm 1.13)\,,
\end{aligned} \tag{23}$$

yielding, respectively,

$$\begin{aligned}
-\frac{1}{2} + \xi_\pi\,I(\xi_\pi) &= -\,0.556 \pm 0.002 + i\,(0.148 \pm 0.029)\,, \\
-\frac{1}{2} + \xi_K\,I(\xi_K) &= -\,0.049 \pm 0.192 + i\,(0.697 \pm 0.027)\,.
\end{aligned} \tag{24}$$

Combining all the previous results, we arrive at

$$\begin{aligned}
& M_{\pi\text{-loop}} = \\
&= (-\,1.383 \pm 0.008 + i\,(0.369 \pm 0.071)) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for}\ \ \phi_s{}' = 0^\circ\,, \\
&= (-\,1.341 \pm 0.037 + i\,(0.357 \pm 0.070)) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for}\ \ \phi_s{}' \simeq +\,(18^\circ \pm 2^\circ)\,, \\
&= (-\,1.341 \pm 0.037 + i\,(0.357 \pm 0.070)) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for}\ \ \phi_s{}' \simeq -\,(18^\circ \pm 2^\circ)\,, \\
& M_{\text{K-loop}} = \\
&= (-\,0.087 \pm 0.343 + i\,(1.247 \pm 0.068)) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for}\ \ \phi_s{}' = 0^\circ\,, \\
&= (-\,0.039 \pm 0.153 + i\,(0.554 \pm 0.173)) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for}\ \ \phi_s{}' \simeq +\,(18^\circ \pm 2^\circ)\,, \\
&= (-\,0.131 \pm 0.514 + i\,(1.869 \pm 0.173)) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for}\ \ \phi_s{}' \simeq -\,(18^\circ \pm 2^\circ)\,, \\
& M_{\pi\text{-loop}} + M_{\text{K-loop}} = \\
&= (-\,1.470 \pm 0.343 + i\,(1.615 \pm 0.099)) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for}\ \ \phi_s{}' = 0^\circ\,, \\
&= (-\,1.379 \pm 0.157 + i\,(0.912 \pm 0.187)) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for}\ \ \phi_s{}' \simeq +\,(18^\circ \pm 2^\circ)\,, \\
&= (-\,1.471 \pm 0.515 + i\,(2.226 \pm 0.186)) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for}\ \ \phi_s{}' \simeq -\,(18^\circ \pm 2^\circ)\,, \\
& |M_{\pi\text{-loop}} + M_{\text{K-loop}}| = \\
&= (2.184 \pm 0.242) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for}\ \ \phi_s{}' = 0^\circ\,, \\
&= (1.653 \pm 0.167) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for}\ \ \phi_s{}' \simeq +\,(18^\circ \pm 2^\circ)\,, \\
&= (2.668 \pm 0.324) \cdot 10^{-2}\ \mathrm{GeV}^{-1} \quad \text{for}\ \ \phi_s{}' \simeq -\,(18^\circ \pm 2^\circ)\,.
\end{aligned} \tag{25}$$

If we again reduce the kaon-loop amplitude by 10% owing to the $\kappa(900)$, and assume for the moment that the $f_0(1370)$ is purely $\bar{n}n$, we get for the modulus of the decay amplitude the value $(2.09 \pm 0.25) \cdot 10^{-2}$ GeV^{-1}, in good agreement with the experimental result in Eq. (17). Taking instead a mixing angle of $\phi_s' = 18° \pm 2°$ produces an amplitude value of $(1.62 \pm 0.17) \cdot 10^{-2}$ GeV^{-1}, also well within the experimental error bars. On the other hand, choosing a negative mixing angle of $\phi_s' = -18° \pm 2°$ gives rise to a somewhat too large amplitude, albeit still compatible with the experimentally allowed range of values, namely $(2.51 \pm 0.34) \cdot 10^{-2}$ GeV^{-1}. So a positive mixing angle seems to be clearly favored. Further increasing a positive ϕ_s' from $+18°$ will yield smaller and smaller amplitudes, until at about $60°$ a minimum is reached of $\approx 0.94 \cdot 10^{-2}$ GeV^{-1}, after which the amplitude increases again. For $\phi_s' > 80°$, there would be agreement again with experiment. However, such a large mixing angle, which would imply an almost pure $\bar{s}s$ substructure for the $f_0(1370)$ seems to be excluded by the weak processes discussed in the previous section, as well as by hadronic decays [5].

Alternatively, if instead we try the $\bar{n}n$ u, d quark loops, the $f_0(1370) \to 2\gamma$ amplitude would be [13], for $\xi \simeq m_u^2/m_{f_0(1370)}^2 \simeq m_d^2/m_{f_0(1370)}^2 \leq 1/4$,

$$
\begin{aligned}
M(f_0(1370) \to 2\gamma) &= \sqrt{2}\ \mathrm{Tr}\,[Q^2\, Q_{\bar{n}n}]\ \frac{\alpha\, N_c}{\pi\, F_\pi}\ 2\,\xi\,[\,2 + (1 - 4\,\xi)\, I(\xi)\,] \\
&= \frac{5\,\alpha\, N_c}{9\,\pi\, F_\pi}\ 2\,\xi\,[\,2 + (1 - 4\,\xi)\, I(\xi)\,]\,.
\end{aligned}
\tag{26}
$$

For $\xi < 1/4$, the values $0.053 < \xi \simeq m_u^2/m_{f_0(1370)}^2 \simeq m_d^2/m_{f_0(1370)}^2 < 0.086$ are compatible with the experimental estimate in Eq. (17), i.e., $|M(f_0(1370) \to 2\gamma)| = (1.90 \pm 0.68) \cdot 10^{-2}$ GeV^{-1}. For $m_{f_0(1370)} \simeq 1370$ MeV, the allowed ranges for $\xi < 1/4$ yield 315.40 MeV $< m_u \simeq m_d <$ 401.76 MeV (see Fig. 1). Using $I(\xi)$ given in Eq. (15), we observe that for all $\xi > 1/4$, which would anyhow imply unrealistically large quark masses, the quark-loop rate is not consonant with the experimental estimate. The allowed range for the constituent u,d mass is quite consistent with the $f_0(1370)$ being purely $\bar{n}n$, or with a small $\bar{s}s$ admixture, of course. On the other hand, taking the $f_0(1370)$ to be mostly $\bar{s}s$, it is almost impossible to find any reasonable quark masses and mixing angles to get agreement with experiment.

4 Conclusions

In this paper we have studied weak and electromagnetic decay processes with isoscalar scalar mesons in the final and initial state, respectively, in order to identify the quark substructure of especially the $f_0(980)$, $f_0(1370)$, and $f_0(1500)$ resonances.

Calculating the weak process $D_s^+ \to f_0 \pi^+$, which has been oberved for the $f_0(980)$, $f_0(1500)$,

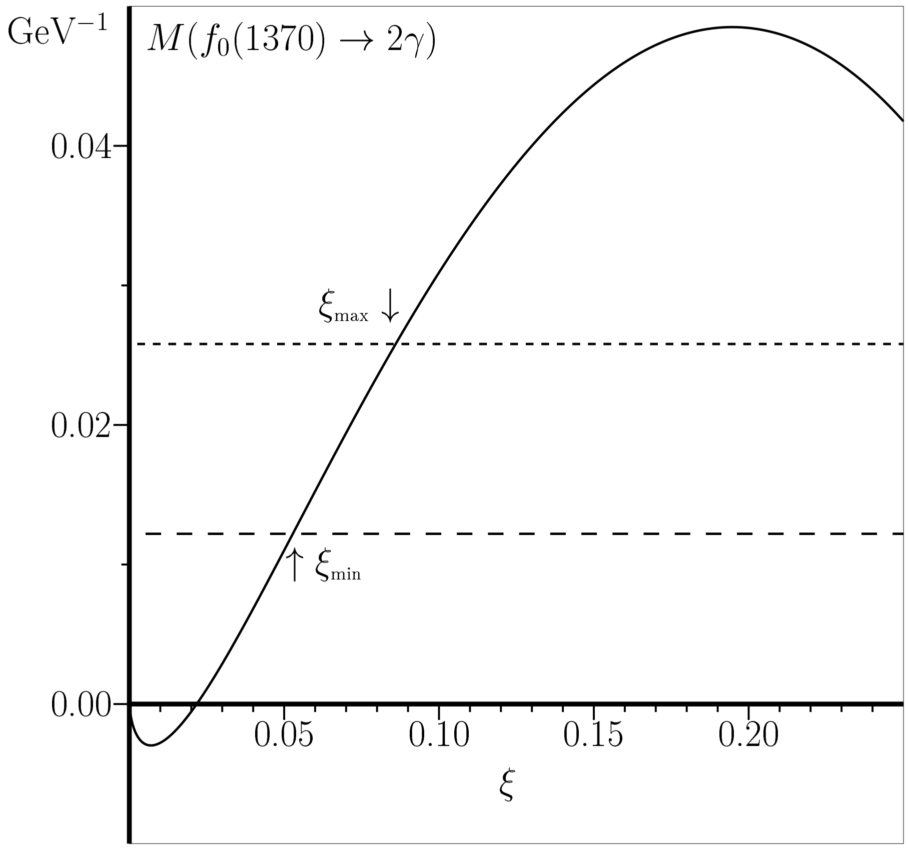

Figure 1: Two-photon-decay amplitude of the $f_0(1370)$ determined by u, d quark loops.

and $f_0(1710)$, via the standard W^+-emission graph, leads to good agreement with experiment for the $f_0(980)$ and $f_0(1500)$, if these states are assumed to be mostly $\bar{s}s$. For the $f_0(1710)$, the large experimental error does not allow a definite conclusion about a possible dominant $\bar{s}s$ configuration, but a mostly $\bar{n}n$ substructure of this resonance is unlikely. As to the $f_0(1370)$, the PDG tables do not report the process $D_s^+ \to f_0(1370)\pi^+$ at all, which would exclude a mostly $\bar{s}s$ nature of this resonance. Not even the observation of the process by the E791 collaboration seems to affect this conclusion, since $D_s^+ \to f_0(1370)\pi^+ \to K^+K^-\pi^+$ is *not* observed [12].

Regarding the electromagnetic processes, calculation of the experimentally observed two-photon decays $f_0(980) \to \gamma\gamma$ and $f_0(1370) \to \gamma\gamma$, using either quark or meson loops, leads to good agreement with the experimentally measured rates, provided that the $f_0(980)$ is assumed to be mostly $\bar{s}s$ and the $f_0(1370)$ mainly $\bar{n}n$. While the quark-loop results depend rather sensitively on the (model-dependent) quark masses and mixing angles, especially in the case of the $f_0(980)$, the meson-loop results only depend on the $\bar{n}n$ vs. $\bar{s}s$ mixing and, therefore, are more stable and reliable.

At this point we should remark that, in a strict SU(3) extension of the quark-level LSM (qlLSM) [17], which to some extent underlied our approach here, both quark *and* meson loops should be included in the two-photon decay amplitude of the $f_0(980)$, being a ground-state scalar meson. As a matter of fact, the contributions of both kinds of loops are needed for the $\sigma(600)$ — in the SU(2) case — so as to get near the not-so-well known experimental two-photon width of the $f_0(400–1200)$ (see Ref. [20], reference no. 19). However, as mentioned in the text, the quark-loop result for the $f_0(980)$ is very sensitive to the quark masses and the mixing angle, due to a rate-enhancement factor of 25 for the nonstrange $\bar{q}q$ component. By a judicious but not unreasonable choice of these parameters, one can easily make the quark-loop contribution vanish, which would occur (using $g_{f_0\,SS} = \sqrt{2}\ 2\pi/\sqrt{3}$) for e.g. $m_{u,d} = 340$ MeV, $m_s = 490$ MeV, $\phi_s = 12.4°$, or $m_{u,d} = 300$ MeV, $m_s = 432$ MeV, $\phi_s = 18.3°$, or all kinds of intermediate values. Therefore, our conclusion on the dominantly $\bar{s}s$ nature of the $f_0(980)$ is upheld no matter which framework is used, i.e., either the rigorous SU(3) qlLSM or the more phenomenological meson-loops-only approach.

Summarizing, weak and electromagnetic processes lend quantitative evidence to a dominantly $\bar{s}s$ interpretation of the $f_0(980)$ and $f_0(1500)$, and a mostly $\bar{n}n$ assignment for the $f_0(1370)$, showing no necessity to consider glueball admixtures.

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$$
\begin{aligned}
|M(K^+ \to \pi^+\pi^0)| &= \frac{G_F\,|V_{ud}|\,|V_{us}|}{2\sqrt{2}}\,F_\pi\,(m^2_{K^+} - m^2_{\pi^0}) = (1.837 \pm 0.020)\cdot 10^{-8}\ \mathrm{GeV} \\
&\qquad\qquad (\text{data [9]: } (1.832 \pm 0.007)\cdot 10^{-8}\ \mathrm{GeV}), \\
|M(D^+ \to \pi^+\pi^0)| &= \frac{G_F\,|V_{ud}|\,|V_{cd}|}{2\sqrt{2}}\,F_\pi\,(m^2_{D^+} - m^2_{\pi^0}) = (28.9 \pm 2.1)\cdot 10^{-8}\ \mathrm{GeV} \\
&\qquad\qquad (\text{data [9]: } (38.6 \pm 5.4)\cdot 10^{-8}\ \mathrm{GeV}), \\
|M(D^+ \to \pi^+\bar{K}^0)| &= \frac{G_F\,|V_{ud}|\,|V_{cs}|}{2}\,F_\pi\,(m^2_{D^+} - m^2_{\bar{K}^0}) = (177 \pm 27)\cdot 10^{-8}\ \mathrm{GeV} \\
&\qquad\qquad (\text{data [9]: } (136 \pm 6)\cdot 10^{-8}\ \mathrm{GeV}).
\end{aligned}
$$

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Note that an erroneous factor $\sqrt{2}$ appeared in the denominator of Eq. (3) of the original paper, instead of the correct factor 2. So the theoretical prediction is reduced by a factor $\sqrt{2}$, being still in good agreement with experiment (see Erratum).

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However, the author interpretes this as an indication of W^+-annihilation amplitudes or strong rescattering in the final state, and also notes that no $f_0(1370)$ is observed in the $D_s^+ \to K^+K^-\pi^+$ final state, pointing to the $f_0(1370)$ being a non-$\bar{s}s$ particle.

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